1) Vectors $\vec{a}$ and $\vec{b}$ are parallel $\iff$ (?)

Vectors $\vec{a}$ and $\vec{b}$ are orthogonal $\iff$ (?)

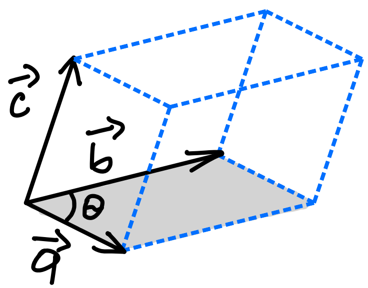
Vectors $\vec{a}, \vec{b}, \vec{c}$ are co-planar $\iff$ (?)

2) $(\vec{a} \times \vec{b}) \cdot \vec{a} =$ (?)

$\vec{a} \times \vec{a} =$ (?)

$\vec{a} \cdot \vec{a} =$ (?)

3)



$\vec{a} \cdot \vec{b} =$

$|\vec{a} \times \vec{b}| =$

$|\vec{a} \cdot (\vec{b} \times \vec{c})| =$

Write the answer in terms of the objects on the picture

4)

$(\vec{a} \cdot \vec{b}) \cdot \vec{c} \stackrel{?}{=} \vec{a} \cdot (\vec{b} \cdot \vec{c})$

$(\vec{a} \times \vec{b}) \times \vec{c} \stackrel{?}{=} \vec{a} \times (\vec{b} \times \vec{c})$

$\vec{a} \cdot (\vec{b} \times \vec{c}) \stackrel{?}{=} (\vec{a} \times \vec{b}) \cdot \vec{c}$

$\vec{a} \cdot \vec{b} \stackrel{?}{=} \vec{b} \cdot \vec{a}$

$\vec{a} \times \vec{b} \stackrel{?}{=} \vec{b} \times \vec{a}$

True or False?

Solutions

1) Vectors $\vec{a}$ and $\vec{b}$ are parallel $\Leftrightarrow$ $\vec{a} \times \vec{b} = 0$

Vectors $\vec{a}$ and $\vec{b}$ are orthogonal $\Leftrightarrow$ $\vec{a} \cdot \vec{b} = 0$

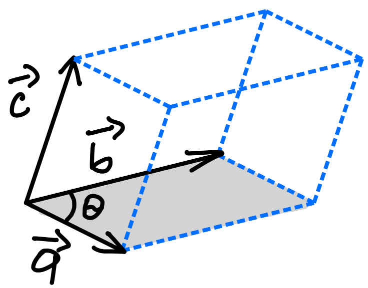
Vectors $\vec{a}, \vec{b}, \vec{c}$ are co-planar $\Leftrightarrow$ $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

2) $$(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$$

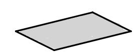
$$\vec{a} \times \vec{a} = 0$$

$$\vec{a} \cdot \vec{a} = |\vec{a}|^2$$

3)



$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ = Area of the "  "

$|\vec{a} \cdot (\vec{b} \times \vec{c})|$ = Volume of the parallelepiped

4)

$$\underbrace{\underbrace{(\vec{a} \cdot \vec{b})}_{\text{scalar}} \cdot \vec{c}}_{\text{does NOT make sense}} \neq \vec{a} \cdot (\vec{b} \cdot \vec{c})$$

$$(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$$

Ex: $(\vec{i} \times \vec{i}) \times \vec{j} = 0 \neq \vec{i} \times (\vec{i} \times \vec{j}) = -\vec{j}$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) \overset{\checkmark}{=} (\vec{a} \times \vec{b}) \cdot \vec{c}$$

$$\vec{a} \cdot \vec{b} \overset{\checkmark}{=} \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a} \qquad \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

Find dot and cross products of the vectors $\vec{a}$ and $\vec{b}$.

1) $\vec{a} = \langle 0, 1, -1 \rangle$; $\vec{b} = \langle 3, 2, 1 \rangle$

2) $\vec{a} = \langle 2, 4, 1 \rangle$; $\vec{b} = \langle 0, 1, -1 \rangle$

3) $\vec{a} = \langle 2, 1, 1 \rangle$; $\vec{b} = \langle 0, 3, -1 \rangle$

4) $\vec{a} = \langle 2, 0, 1 \rangle$; $\vec{b} = \langle 0, 3, -1 \rangle$

5) $\vec{a} = \langle 2, 0, 1 \rangle$; $\vec{b} = \langle 1, 2, -1 \rangle$

6) $\vec{a} = \langle 2, 1, 0 \rangle$; $\vec{b} = \langle 2, 1, -1 \rangle$

Solutions

Find dot and cross products of the vectors $\vec{a}$ and $\vec{b}$.

$$\boxed{\vec{a} = \langle 2, 1, 0 \rangle \; ; \; \vec{b} = \langle 2, 1, -1 \rangle}$$

$$\vec{a} \cdot \vec{b} = 2(2) + 1(1) + 0(-1) = 4 + 1 + 0 = 5$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ 2 & 1 & -1 \end{vmatrix} = \langle -1 - 0, \; 0 + 2, \; 2 - 2 \rangle = \langle -1, \; 2, \; 0 \rangle$$

$$\left| \begin{array}{ccc|ccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 & 2 & 1 & 0 \\ 2 & 1 & -1 & 2 & 1 & -1 \end{array} \right|$$

$$\vec{i}(1)(-1) + \vec{j}(0)(2) + \vec{k}(2)(1)$$

$$- \vec{i}(0)(1) - \vec{j}(2)(-1) - \vec{k}(1)(2)$$

$$= \vec{i}\big(1(-1) - 0(1)\big) + \vec{j}\big(0(2) - 2(-1)\big) + \vec{k}\big(2(1) - 1(2)\big)$$

$$= -\vec{i} + 2\vec{j} = \langle -1, 2, 0 \rangle$$

$\vec{a} = \langle 0, 1, -1 \rangle$; $\vec{b} = \langle 3, 2, 1 \rangle$

$\vec{a} \cdot \vec{b} = 0(3) + 1(2) + (-1)(1) = 1$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix} = \langle 1+2, -3-0, 0-3 \rangle$$

$$= \langle 3, -3, -3 \rangle$$

$\vec{a} = \langle 2, 4, 1 \rangle$; $\vec{b} = \langle 0, 1, -1 \rangle$

$\vec{a} \cdot \vec{b} = 2(0) + 4(1) + 1(-1) = 3$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \langle -4-1, 0+2, 2-0 \rangle$$

$$= \langle -5, 2, 2 \rangle$$

$\vec{a} = \langle 2, 1, 1 \rangle$; $\vec{b} = \langle 0, 3, -1 \rangle$

$\vec{a} \cdot \vec{b} = 2(0) + 1(3) + 1(-1) = 2$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 0 & 3 & -1 \end{vmatrix} = \langle -1-3, 0+2, 6-0 \rangle$$

$$= \langle -4, 2, 6 \rangle$$

$\vec{a} = \langle 2, 0, 1 \rangle$; $\vec{b} = \langle 0, 3, -1 \rangle$

$\vec{a} \cdot \vec{b} = 2(0) + 0(3) + 1(-1) = -1$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 1 \\ 0 & 3 & -1 \end{vmatrix} = \langle 0-3, 0+2, 6-0 \rangle$$

$$= \langle -3, 2, 6 \rangle$$

$\vec{a} = \langle 2, 0, 1 \rangle$; $\vec{b} = \langle 1, 2, -1 \rangle$

$\vec{a} \cdot \vec{b} = 2(1) + 0(2) + 1(-1) = 1$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 1 \\ 1 & 2 & -1 \end{vmatrix} = \langle 0-2, 1+2, 4-0 \rangle$$

$$= \langle -2, 3, 4 \rangle$$

$\vec{a} = \langle 2, 1, 0 \rangle$; $\vec{b} = \langle 2, 1, -1 \rangle$

$\vec{a} \cdot \vec{b} = 2(2) + 1(1) + 0(-1) = 5$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ 2 & 1 & -1 \end{vmatrix} = \langle -1+0, 0+2, 2-2 \rangle$$

$$= \langle -1, 2, 0 \rangle$$